UNIT-3 DSTL

Hasse Diagram :- A Hasse diagram is a graphical representation of the relation of elements of a partially ordered set (poset) with an implied upward orientation.

→ A point is drawn for each element of the partially ordered set (poset) and Joined with the line segment according to the following rules:

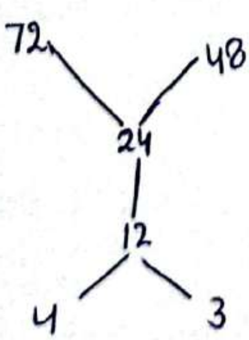
- If $p < q$ in the poset, then the point corresponding to p appears lower in the drawing than the point corresponding to q.
- The two points p and q will be Joined by line segment if p is related to q.

⇒ To draw a Hasse diagram, provided set must be a poset.

A poset or partially ordered set A is a pair, $(B, \leq)$ of a set B whose elements are called the vertices of A and obeys the following rules:

1. Reflexivity $p \leq p \quad \forall p \in B$
2. Anti-symmetric - $p \leq q$ and $q \leq p$ if $p = q$
3. Transitivity - If $p \leq q$ and $q \leq r$ then $p \leq r$

Ex: (1) Draw Hasse diagram for $(\{3, 4, 12, 24, 48, 72\}, /)$



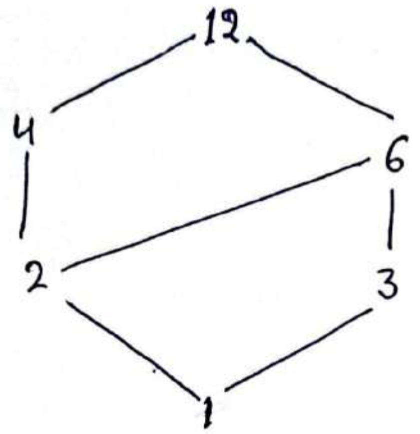
POSET

$A = \{(3<12), (3<24), (3<48), (3<72), (4<12), (4<24), (4<48), (4<72), (12<24), (12<48), (12<72), (24<48), (24<72)\}$

Ex②: Draw Hasse Diagram for (D12, /)

D12 = { 1, 2, 3, 4, 6, 12 }

poset A = { (1<2), (1<3), (1<4), (1<6), (1<12), (2<4), (2<6), (2<12), (3<6), (3<12), (4<12), (6<12) }



UNIT-3

Lattices :- Definition, Properties of lattices - Bounded, Complemented, Modular and Complete lattice, **Boolean Algebra**: Introduction, Axioms and Theorems of Boolean Algebra, Algebraic manipulation of Boolean expressions. Simplification of Boolean functions, Karnaugh maps, logic gates, Digital circuits and Boolean Algebra.

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finite set less than or equal to

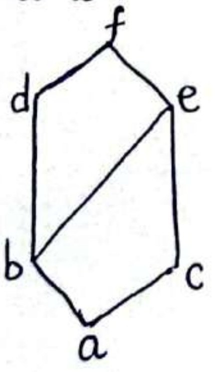
Lattices:- A lattice is a poset in $(L, \leq)$ in which every subset $\{a, b\}$ consisting of two elements has a least upper bound (LUB) and a greatest lower bound. (GLB)

LUB $(\{a,b\})$ (Supremum) is denoted by $a \vee b$ and is called the Join of a and b.

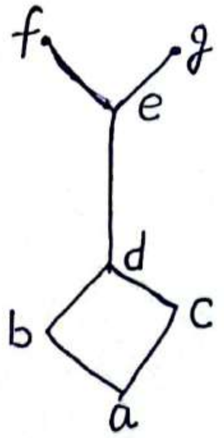
GLB $(\{a,b\})$ (Infemum) is denoted by $a \wedge b$ and is called the meet of a and b.

~~a) is a la~~

Ex:-



(a)



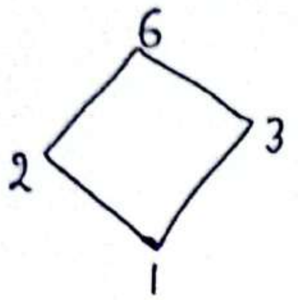
(b)

(a) is a lattice

(b) is not a lattice because $f \vee g$ does not exist.

$$\boxed{\begin{array}{l} \text{GLB } \{a,b\} = a \wedge b \text{ (or) } a * b \text{ (meet or product of a and b)} \\ \text{LUB } \{a,b\} = a \vee b \text{ (or) } a \oplus b \text{ (Join or sum of a and b)} \end{array}}$$

Ex:- Check whether following Hasse diagram of poset is a Lattice or Not.



union

LUB Table

∨	1	2	3	6
1	1	2	3	6
2	2	2	6	6
3	3	6	3	6
6	6	6	6	6

Intersection

GLB Table

∧	1	2	3	6
1	1	1	1	1
2	1	2	1	2
3	1	1	3	3
6	1	2	3	6

∵ for every pair (a,b) of poset, both LUB exists and GLB exist.

∴ POSET is a lattice.

Thm:

If $(L1, \leq)$ and $(L2, \leq)$ are lattices then $(L, \leq)$ is a lattice where $L = L1 \times L2$ and the partial order $\leq$ of L is the product of partial order.

II. Properties of Lattices

1. Idempotent Properties
   a) $a \vee a = a$
   b) $a \wedge a = a$
2. Commutative Properties
   a) $a \vee b = b \vee a$
   b) $a \wedge b = b \wedge a$
3. Associative Properties
   a) $a \vee (b \vee c) = (a \vee b) \vee c$
   b) $a \wedge (b \wedge c) = (a \wedge b) \wedge c$
4. Absorption Properties
   a) $a \vee (a \wedge b) = a$
   b) $a \wedge (a \vee b) = a$

Theorem

a) $a \vee b = b$ iff $a \leq b$

b) $a \wedge b = a$ iff $a \leq b$

c) $a \wedge b = a$ iff $a \vee b = b$

Theorem

1. If $a \leq b$ then
   a) $a \vee c \leq b \vee c$
   b) $a \wedge c \leq b \wedge c$

2. $a \leq c$ and $b \leq c$ iff $a \vee b \leq c$

3. $c \leq a$ and $c \leq b$ iff $c \leq a \wedge b$

4. If $a \leq b$ and $c \leq d$ then
   a) $a \vee c \leq b \vee d$
   b) $a \wedge c \leq b \wedge d$

(iii) Types of Lattices

1. Isomorphic Lattices

If $f: L1 \rightarrow L2$ is an isomorphism from the poset $(L1, \leq 1)$ to the poset $(L2, \leq 2)$ then L1 is a lattice iff L2 is a lattice.

If a and b are elements of L1 then

$\left\{ \begin{array}{l} \text{(one-one \& onto)} \\ \text{Bijection from } L_1 \text{ to } L_2 \\ f: L_1 \rightarrow L_2 \end{array} \right\}$

$f(a \wedge b) = f(a) \wedge f(b)$ and

$f(a \vee b) = f(a) \vee f(b)$

If two lattices are isomorphic as posets we say they are isomorphic lattices.

2. Bounded Lattice:

A Lattice is said to be bounded if it has a greatest element 1 and a least element 0. Ex:

3. Complemented Lattice

A lattice L is said to be Complemented if it is bounded and if every element in L has a Complement.

Theorem :

Let $L = \{a_1, a_2, a_3, a_4, \ldots a_n\}$ be a finite lattice. The L is bounded.

Th^m :-

Let L be bounded lattice with greatest Element I and least element 0 and let a belong to L. an Element a' belong to L is a Complement of a if

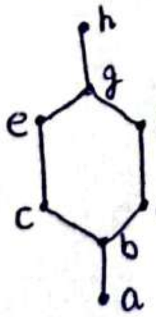
$$a \vee a' = I \quad \text{and} \quad a \wedge a' = 0$$

Theorem :

Let L be a bounded distributive lattice. If Complement exists it is unique.

Example :-

Ex ① If the given Hasse diagram a lattice?



Solution :- a Hasse diagram is called a lattice if it is both meet semilattice and Join semilattice.

i.e.

$\forall x, y \in L$, $GLB(x, y) \neq \phi$ and

$\forall x, y \in L$, $LUB(x, y) \neq \phi$

Consider the incomparable pairs. f, e ; c, d ; e, d

~~e, f are in~~

$GLB(f,e) = b$
$LUB(f,e) = g$

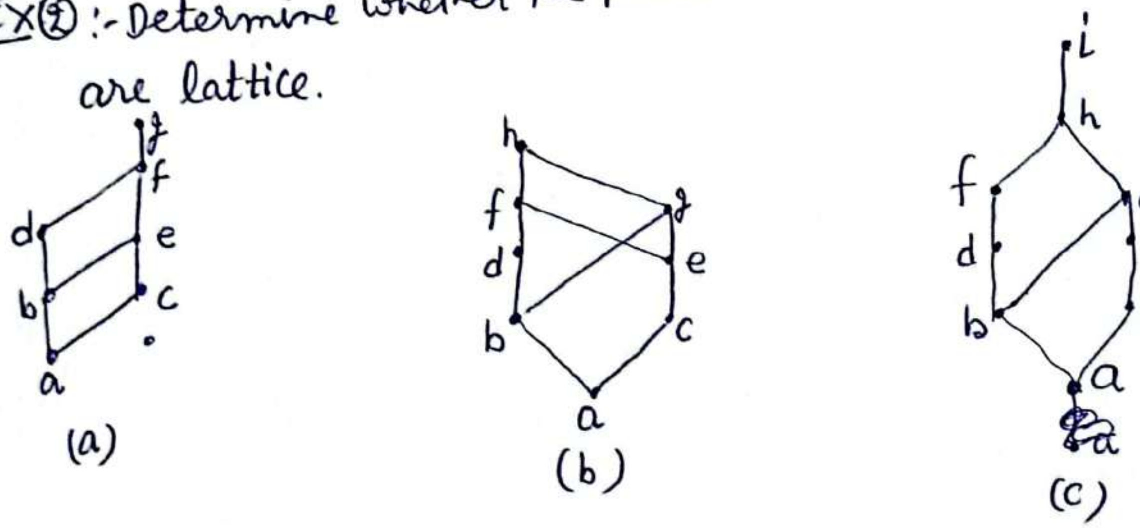
$GLB(c,d) = b$
$LUB(c,d) = g$

$GLB(e,d) = b$
$LUB(e,d) = g$

$GLB(c,f) = b$
$LUB(c,f) = g$

Since all incomparable pairs have GLB and LUB so given Hasse diagram is a lattice.

Ex②:- Determine whether the posets with these given Hasse diagrams are lattice.



Sol^n.(a) (d, e), (b, c), } Lattice

Sol^n(b) (f, g)

$GLB(f,g) = \phi$
$LUB(f,g) = h$
} - The given Hasse diagram is not a meet semilattice and hence, it is not a lattice.

Sol^n.- (f, g), (d, e) } Lattice.

Ex: Determine whether these posets are lattices.

a) $(\{1,3,6,9,12\}, |)$

b) $(\{1,5,25,125\}, |)$

c) $(Z, \geq)$

d) $(P(S), \supseteq)$

Types of Lattices

① **Isomorphic Lattice**:- If there exists an isomorphism between two lattices, then lattices are called isomorphic.

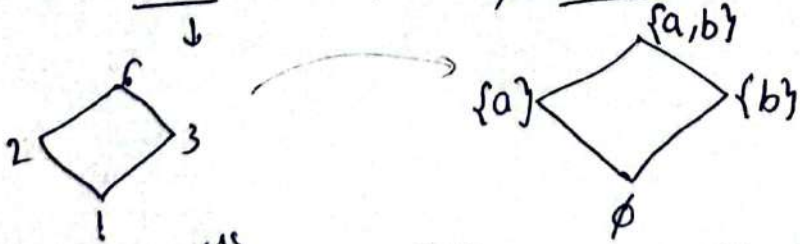
$f: L_1 \rightarrow L_2$

Ex:- Let $L=\{1,2,3,6\}$ and $A=\{a,b\}$. Then prove that the lattices $(L, /)$ and $(P(A), \subseteq)$ are isomorphic.

Sol^n. Consider the mapping $f: L \rightarrow P(A)$, where $P(A)=\{\phi, \{a\}, \{b\}, \{a,b\}\}$ and the mapping.

$f(1)=\phi$, $f(2)=\{a\}$, $f(3)=\{b\}$, $f(6)=\{a,b\}$.

Hasse diagram for L.   for P(A)



Condition (i) L.H.S R.H.S

$f(a \wedge b) = f(a) \wedge f(b)$:

let $a=1$ $b=2$.

$f(1 \wedge 2) = f(1) \wedge f(2)$

$f(1) = \phi \wedge \{a\}$

$\phi. = \phi$ ✓ True

Condition (ii) L.H.S R.H.S.

$f(a \vee b) = f(a) \vee f(b)$

$f(1 \vee 2) = f(1) \vee f(2)$

$\Rightarrow f(2) = \phi \vee \{a\}$

$\{a\} = \{a\}$.

True.

--- Since both properties are holding for all pairs

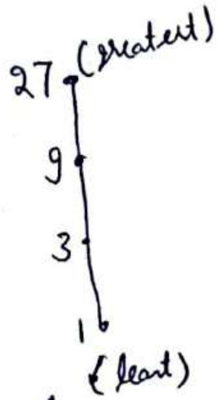
So: function is isomorphism.

→ So. both lattices are isomorphic.

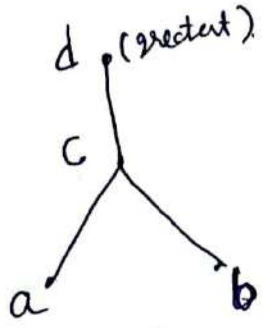
(2) Bounded Lattice :-

A lattice which has both elements least and greatest (denoted by 0 and 1 respectively) is called a bounded lattice.

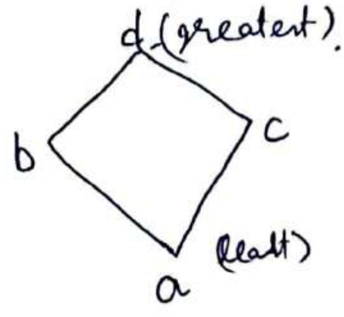
Ex:



↓ Bounded



↓ Not bounded



↓ bounded.

(3) Complemented Lattice :-

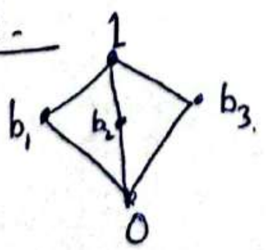
In a bounded lattice $(L, \vee, \wedge, 0, 1)$ (0 = least elem, 1 = greatest element) an element $b \in L$ is called a complement of an element $a \in L$ if.

$$a \wedge b = 0 \quad \text{and} \quad a \vee b = 1.$$

where 0 and 1 are lower and upper bound of L.

A bounded lattice is said to be complemented if every element has at least one complement in the lattice.

Ex:



(b_1, b_2)	(b_2, b_3)	(b_1, b_3)
$\wedge \to 0$	$\wedge \to 0$	$\wedge \to 0$
$\vee \to 1$	$\vee \to 1$	$\vee \to 1$

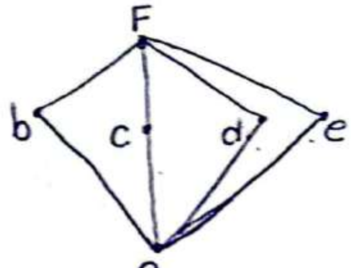
Boolean Algebra

① **Def^n:-** A lattice is said to be a B.A. if it is both distributive & Complemented → "Every element has at ~~most~~ least one Complement."

↳ "Every element has at most one Complement."

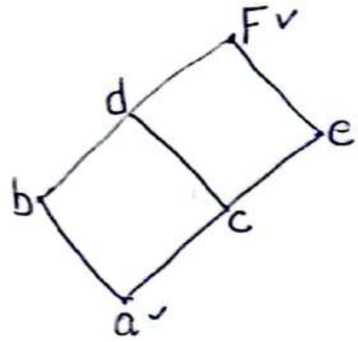
⇒ A lattice is said to be a B.A. if it's every element has exactly one complement.

Ex:-



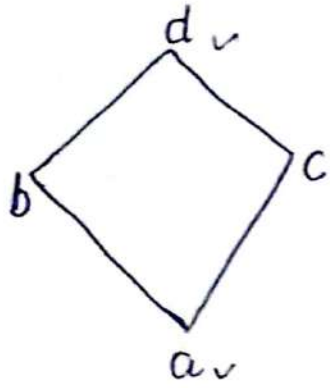
$b^c = c = d = e$

Since b has more that one Completment so not B.A.



$b^c = e$

since c doesnot have any Coplemnt so it is not a B.A.



$a^c = d$

$d^c = a$

$b^c = c$

Since every elemnt has exactly one Coplemnt so it is B.A.

② **Def^n:-** Let B be a non-empty set with

- two binary operations + & ·
- one unary operation ' Complement
- two distinct elements 0 & 1

$(B, +, \cdot, ', 0, 1)$ ↳ least

Properties:-

① closure law

$\forall\, a, b \in B \quad a + b \in B$

$\qquad\qquad\quad a \cdot b \in B$

(2) Associative law:-

$$(a+b)+c = a+(b+c)$$

$$(a \cdot b) \cdot c = a \cdot (b \cdot c)$$

(3) Identity law:-

$$a + \overset{0}{?} = a$$

$$a \cdot \underset{1}{?} = a$$

(4) Complemented law

$$a + \bar{a} = 1$$

$$a \cdot \bar{a} = 0$$

(5) Commutative law

$$a+b = b+a$$

$$a \cdot b = b \cdot a$$

(6) Distributive law.

$$a + (b \cdot c) = (a+b) \cdot (a+c)$$

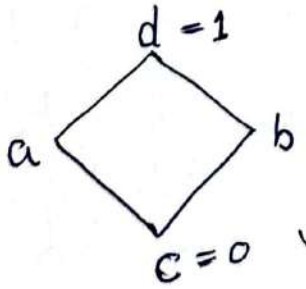
$$a \cdot (b+c) = a \cdot b + a \cdot c$$

If a set satisfying all above properties so, it is known as Boolean Algebra.

Note:- These properties are related with lattice.

as:- $+ = \vee \rightarrow$ Join
$\cdot = \wedge \rightarrow$ Meet

Ex:-



$B = \{a, b, c, d\}$

$\rightarrow$ All above propeties satisfy.

Theorems of B. A.

Prove Thm ①

(A) Idempotent Laws

1) $\forall a \in B,\ a+a=a$

2) $\forall a \in B,\ a.a=a$

Proof:- (1)

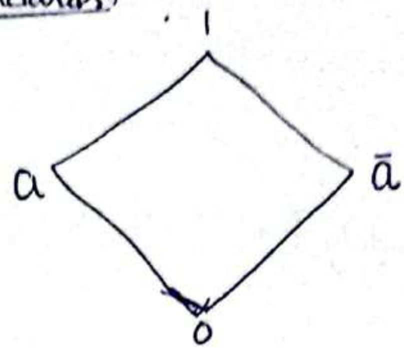
L.H.S

$$a+a = (a+a).\underline{1}$$
$$=(a+a).(a+\bar{a})$$
$$= a+(a.\bar{a})$$ -- Distributive Law
$$= a+0$$
$$= a$$ Proved.

Standards for Prove of Thm.

$+ = \vee$

$. = \wedge$



results

① $a+\bar{a}=1$ ② $a.\bar{a}=0$

③ $a+0=a$ ④ $a.1=a$

⑤ $a+1=1$ ⑥ $a.0=0$

⑦ .1 ⑧ +0

Proof (2)

L.H.S

$$a.a = (a.a)+\underline{0}$$
$$= (a.a)+(a.\bar{a})$$
$$= a.(a+\bar{a})$$
$$= a.1$$
$$= a$$ Proved.

Thm: ② Prove that elements '0' & '1' are unique.

Proof: Let $0_1, 0_2$ be two elements.

If 0_1 is l.b. (lower bound)

$$a+0_1 = a$$
$$0_2+0_1 = 0_2 \quad -- ①$$

If 0_2 is l.b.

$$a+0_2 = a$$
$$0_1+0_2 = 0_1 \quad -- ②$$

by using ① and ②

$$0_1 = 0_2$$

Let $1_c, 1_d$ be two elements

If 1_c is U.B. (upper bound)

$$a.1_c = a$$
$$1_d.1_c = 1_d \quad — ①$$

If 1_d is U.B. (upper bound)

$$a.1_d = a$$
$$1_c.1_d = 1_c \quad - ②$$

by using ① & ②

$$1_c = 1_d.$$ Proved

Th^m 3:- Prove that element '0' & '1' are ~~distributed~~ distinct and

$\bar{0} = 1, \quad \bar{1} = 0$

Proof:- suppose $0 = 1$

from standard diagram & result 4 & 6.

$a \cdot 1 = a$
$a \cdot 0 = 0$

$\Rightarrow \boxed{a = 0}$

so $0 \cdot 1 = 0$
$0 \cdot 0 = 0$

In boolean algebra at least two distinct elements will be definitely exist.

so assumption $0 = 1$ is false.

$\Rightarrow \quad 0 \neq 1.$

$\bar{0} = \bar{0} \underbrace{+ 0}_{\text{by rule 8}} = 1$
$\bar{1} = \bar{1} \underbrace{. 1}_{\text{by rule 7}} = 0$

} Proved.

$Th^m 4$:- $\forall a \in B$, Prove that $a + 1 = 1$ and $a \cdot 0 = 0$

Proof:- L.H.S.

$a + 1 = (a+1) . 1$
$= (a+1) . (a + \bar{a})$
$= a + (1 . \bar{a})$
$= a + \bar{a} = 1$

Proof

L.H.S. $a . 0 = (a.0) + 0$
$= (a.0) + (a.\bar{a})$
$= a.(0 + \bar{a})$
$= a.\bar{a} = 0$ Proof.

Th^m 5: Absorption laws

$\forall a, b \in B$ P.T. $a + a \cdot b = a$

$a \cdot (a+b) = a$

Concept of duality

$\wedge \leftrightarrow \vee$

$\vee \leftrightarrow \wedge$

$\leq \leftrightarrow \geq$

$\cdot \longrightarrow +$

$+ \longrightarrow \cdot$

L. H. S.

$$a + a \cdot b = a \cdot 1 + a \cdot b$$
$$= a \cdot (1+b)$$
$$= a \cdot 1$$
$$= a \quad \text{Proved.}$$

for 2nd proof.

using principal of duality

$$a \cdot (a+b) = a$$

$$\Rightarrow a + (a \cdot b) = a.$$ it is already proved.

Th^m 6:- Involution law.

$\forall a \in B$ P.T. $(\overline{\bar{a}}) = a$

we have to find $\bar{a}$

$a + \bar{a} = 1 \qquad a \cdot \bar{a} = 0$

a is Complement of $\bar{a}$

$$a = (\overline{\bar{a}})$$

Proved

Th^n: Prove that in a B.A. Complement of every element is unique.

OR $\forall\ a \in B,\ \exists$ unique $\bar{a}$.

let $a \in B$

$a_1 \ ,\ a_2$

O

$$a + a_1 = 1 = a + a_2$$
$$a \cdot a_1 = 0 = a \cdot a_2$$

$$\begin{aligned} a_1 &= a_1 \cdot 1 \\ &= a_1 \cdot (a + a_2) \\ &= a_1 \cdot a + a_1 \cdot a_2 \\ &= 0 + a_1 \cdot a_2 \\ &= a_1 \cdot a_2 \quad (i) \end{aligned}$$

$$\begin{aligned} a_2 &= a_2 \cdot 1 \\ &= a_2 \cdot (a + a_1) \\ &= a_2 \cdot a + a_2 \cdot a_1 \\ &= 0 + a_2 \cdot a_1 \\ &= a_2 \cdot a_1 \quad (ii) \end{aligned}$$

By using ~~eq~~ (i) & (ii)

$$\boxed{a_1 = a_2}$$

Th^n: De Morgan's Law $\rightarrow$ For any a, b in B.A, P.T

(i) $(a+b)' = a' \cdot b'$ (ii) $(a \cdot b)' = a' + b'$

Proof (i) To prove complement of $a+b$ is $a' \cdot b'$. we will prove that $(a+b) + (a' \cdot b') = 1$ & $(a+b) \cdot (a' \cdot b') = 0$

L.H.S. $(a+b) + (a' \cdot b')$

distributive laws

$$\begin{aligned} &= (a + b + a') \cdot (a + b + b') \\ &= (b + a + a') \cdot (a + b + b') \\ &= (b + 1) \cdot (a + 1) \quad \rightarrow \text{~~Identical~~} \\ &= (1) \cdot (1) \quad \rightarrow \text{Idempotent Law.} \\ &= 1 \end{aligned}$$

Proved

L.H.S $(a+b) \cdot (a' \cdot b') = (a \cdot a' \cdot b') + (b \cdot a' \cdot b')$

$$\begin{aligned} &= (a \cdot a' \cdot b') + (b \cdot b' \cdot a') \\ &= (0 \cdot b') + 0 \cdot 0') \\ &= 0 + 0 = 0 \quad (R.L.) \end{aligned}$$

Proved

(ii)

Proof: To prove Complement of $a \cdot b$ is $a' + b'$

we will P.T.
$$(a \cdot b) + (a' + b') = 1 \ \& \ (a \cdot b) \cdot (a' + b') = 0$$

L.H.S. $(a \cdot b) + (a' + b') = [a + (a' + b')] \cdot [b + (a' + b')]$

$$= [a + a' + b'] \cdot [b + b' + a']$$
$$= [1 + b'] \cdot [1 + a']$$
$$= 1 \cdot 1 = 1$$

$$(a \cdot b) \cdot (a' + b') = [(a \cdot b) \cdot a'] + [a \cdot b \cdot b']$$
$$= [b \cdot a \cdot a'] + [a \cdot b \cdot b']$$
$$= [b \cdot 0] + [a \cdot 0]$$
$$= 0 + 0 = 0$$

Boolean Functions → A function $f: A^n \to A$ is called Boolean function, if it can be specified by a Boolean Expression of 'n' variables.

- Let $A = \{x_1, x_2, \dots, x_n\}$
- & three operations $+, \cdot, '$

$$\left.\begin{array}{l} \cdot\ (x_1 + x_2' \cdot x_3)' + x_2 \cdot x_1 \\ \text{OR} \quad (x_1 \vee (x_2' \wedge x_3) \vee (x_2 \wedge x_1)) \end{array}\right\} \to \text{Boolen expression}$$

Boolean Algebra :- is a branch of algebra in which value of variables are truth values T/F usually denoted by 1 or 0.

x_1	$\bar{x}_1$
0	1
1	0

x_1	x_2	$x_1 \overset{+}{\vee} x_2$
0	0	0
0	1	1
1	0	1
1	1	1

x_1	x_2	$x_1 \overset{\cdot}{\wedge} x_2$
0	0	0
0	1	0
1	0	0
1	1	1

$$B : A^n \to A$$

Boolean function $f(x_1, x_2) = x_1 \vee x_2$

Equivalence of Boolean functions :-

Show that Boolean function $f_1 = (x_1 \vee x_2) \vee x_3$

$f_2 = x_1 \vee (x_2 \vee x_3)$

are equivalent.

x_1	x_2	x_3	$x_1 \overset{+}{\vee} x_2$	$x_2 \overset{+}{\vee} x_3$	f_1	f_2
0	0	0	0	0	0	0
0	0	1	0	1	1	1
0	1	0	1	1	1	1
0	1	1	1	1	1	1
1	0	0	1	0	1	1
1	0	1	1	1	1	1
1	1	0	1	1	1	1
1	1	1	1	1	1	1

So $f_1 = f_2$

f_1 & f_2 are equivalent.

Types of Boolean functions

(a) **POS Form** (Product of Sum Form)

$(\bar{x}_1 + x_2 + x_3) \cdot (\bar{x}_1 + \bar{x}_2 + x_3) \cdot (x_1 + x_2 + x_3)$

OR

$(\bar{x}_1 \vee x_2 \vee x_3) \wedge (\bar{x}_1 \vee \bar{x}_2 \vee x_3) \wedge (\underbrace{x_1 \vee x_2 \vee x_3}_{\text{max term}})$

Conjuctive Normal form (C.N.F)

A boolean Expression is said to be C.NF if it is meet of maxterms.

(b) **SOP Form** (Sum of Product form)

$(\bar{x}_1 \cdot x_2 \cdot x_3) + (\bar{x}_1 \cdot \bar{x}_2 \cdot x_3) + (x_1 \cdot x_2 \cdot x_3)$

OR

$(\bar{x}_1 \wedge x_2 \wedge x_3) \vee (\bar{x}_1 \wedge \bar{x}_2 \wedge x_3) \vee (\underbrace{x_1 \wedge x_2 \wedge x_3}_{\text{min term.}})$

Disjunctive Normal form (D.N.F.)

A Boolean expression is said to be D.N.F. if it is Join of min-terms.

Q^n:- let $f = (\bar{x} \wedge z) \vee y$. Write f in DNF. OR min term normal form

x	y	z	$\bar{x}$	$\bar{x} \wedge z$	f		
0	0	0	1	0	0	m_1	
0	0	1	1	1	1	m_2	$\bar{x} \wedge \bar{y} \wedge z$
0	1	0	1	0	1	m_3	$\bar{x} \wedge y \wedge \bar{z}$
0	1	1	1	1	1	m_4	$\bar{x} \wedge y \wedge z$
1	0	0	0	0	0	m_5	
1	0	1	0	0	0	m_6	
1	1	0	0	0	1	m_7	$x \wedge y \wedge \bar{z}$
1	1	1	0	0	1	m_8	$x \wedge y \wedge z$

Values having 1 are min terms

m_1, m_5, m_6 are max t

DNF $\Rightarrow (\bar{x} \wedge \bar{y} \wedge z) \vee (\bar{x} \wedge y \wedge \bar{z}) \vee (\bar{x} \wedge y \wedge z) \vee (x \wedge y \wedge \bar{z}) \vee (x \wedge y \wedge z)$

CNF $\Rightarrow (x \vee y \vee z) \wedge (\bar{x} \vee y \vee z) \wedge (\bar{x} \vee y \vee \bar{z})$ ← max term normal form

1) Simplifying

$$Z = \bar{A}BC + AB\bar{C} + \bar{A}\bar{B}\bar{C} + A\bar{B}\bar{C} + ABC$$

$$= BC(\bar{A}+A) + \bar{B}\bar{C}(A+\bar{A}) + A\bar{B}C$$

$$= BC + \bar{B}\bar{C} + A\bar{B}C$$

$$= BC + \bar{B}(\bar{C} + AC)$$

Absorbtion Law $\{x + \bar{x}y = x + y\}$ $\{\bar{x} + xy = \bar{x} + y\}$

$$= BC + \bar{B}(\bar{C} + A)$$

$$= BC + \bar{B}\bar{C} + A\bar{B}$$

So $\boxed{Z = BC + \bar{B}\bar{C} + A\bar{B}}$

$(BC + \bar{B}\bar{C} = 1)$ ✗ individuals variables are complemented.

2) Simplify

$$Z = AB + \overline{AC} + A\bar{B}C\,(AB + C)$$

$$= AB + \overline{AC} + A\bar{B}CAB + A\bar{B}C.C$$

$$\qquad\qquad\qquad\quad 0$$

$(x.\bar{x} = 0)$ $(x.x = x)$

$$= AB + \overline{AC} + 0 + A\bar{B}C$$

$$= AB + \overline{AC} + A\bar{B}C$$

$$= A(B + \bar{B}C) + \overline{AC}$$

$$= A(B + C) + \overline{AC}$$

(Absorbtion Law)

$$= AB + AC + \overline{AC}$$

$x + \bar{x} = 1$

$$= AB + 1$$

$1 + x = 1$

$$= 1 \checkmark$$

$\boxed{Z = 1}$

3) ~~Simplify~~ :-

~~POS~~ SOP Form Simplification :-

Ex: $F = \bar{A}\bar{B}\bar{C}\bar{D} + A\bar{B}\bar{C}D + AB\bar{C}\bar{D} + ABCD + \bar{A}BCD$

0000   1001   1100   1111   0111

$$F = \Sigma(0, 9, 12, 15, 7)$$

4-variable K-Map

AB \ CD	00	01	11	10
00	1 (0)	0 (1)	0 (3)	0 (2)
01	0 (4)	0 (5)	1 (7)	0 (6)
11	1 (12)	0 (13)	1 (15)	0 (14)
10	0 (8)	1 (9)	0 (11)	0 (10)

Write the Eqn.

$= BCD + \bar{A}\bar{B}\bar{C}\bar{D} + ABC\bar{D}\bar{D} + A\bar{B}\bar{C}D$

Ex:-② $\bar{A}\bar{B}\bar{C} + \bar{A}B\bar{C} + \bar{A}BC + A\bar{B}C$

000 010 011 101 A

$\Sigma m(0,2,3,5)$ / 3 variable k-map

A \ BC	00	01	11	10
0	1 (0)	(1)	1 (3)	1 (2)
1	(4)	1 (5)	(7)	(6)

$A\bar{B}C + \cancel{\bar{A}\bar{B}\bar{C}} + \bar{A}B$

$\bar{A}\bar{B}\bar{C}$

Qn:- $F = \bar{A}\bar{B}C + \bar{A}BC + \bar{A}B\bar{C} + ABC + AB\bar{C}$

001 011 010 111 110

A \ BC	00	01	11	10
0	(0)	1 (1)	1 (3)	1 (2)
1	(4)	(5)	1 (7)	1 (6)

$\Rightarrow B + \bar{A}C$ w +

Qn.-